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A new mechanism for the generation of primordial seeds for the cosmic magnetic fields

M. Gasperini

Dipartimento di Fisica, Università di Bari, Via G. Amendola 173, 70126 Bari, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Bari, Italy

We discuss the inflationary production of magnetic seeds for the galactic dynamo through the photon-graviphoton mixing typical of extended models of local supersymmetry. An analisys of the allowed region in parameter space shows that such a mechanism is compatible with existing phenomenological bounds on the vector mass and on the mixing parameter.

The parametric amplification of the vacuum fluctuations [1], induced by inflation, is one of the most appealing mechanisms [2] for the spontaneous generation of the large-scale magnetic fields required to seed the galactic dynamo, or the galactic magnetic field itself [3]. Unfortunately, the minimal coupling of photons to a four-dimensional geometry is conformally invariant: in that case, the electromagnetic fluctuations are unaffected by the time evolution of a conformally flat metric (typical of the inflationary scenario) and then, in particular, cannot be amplified.

Up to now, known attempts to generate large enough magnetic seeds from the vacuum try to break conformal invariance either at the classical level, through some *ad-hoc* nonminimal coupling of photons to the background curvature [2], or at the quantum leel, though the so-called "trace-anomaly" [4]. Alternatively, it is possible to exploit the nonminimal coupling of photons to a scalar field, the inflaton [5] or the dilaton [6], which cannot be eliminated by a conformal rescaling of the metric, and which plays the role of the external "pump field" amplifying the electromagnetic fluctuations. The coupling to a dynamical dilaton, in particular, is naturally provided by superstring theory, and may act efficiently in the context of the pre-big bang scenario [6].

The physical origin of the conformal invariance, which prevents the inflationary amplification of minimally coupled electromagnetic fluctuations, is the unbroken gauge symmetry of the Maxwell action, leading to massless photons. Even if massless, however, the photon field A_μ could be non-trivially mixed with the massive vector component V_μ of the gravitational supermultiplet – the so-called "graviphoton" – according to the effective action:

$$S = \int \frac{d^4x}{8\pi}\sqrt{-g}\,(-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}G_{\mu\nu}G^{\mu\nu} + \alpha F_{\mu\nu}G^{\mu\nu} + m^2 V_\mu V^\mu), \tag{1}$$

where $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$, $G_{\mu\nu} = 2\partial_{[\mu}V_{\nu]}$. Such a gauge-invariant mixing, parametrized by the dimensionless coefficient α ($|\alpha| < 1$ to avoid unphysical states in the mass spectrum), is indeed required by local supersymmetry in $N = 2$ extended models for the graviphoton [7], and may also arise naturally in the context of higher-dimensional gauge interactions [8]. The above action also describes, formally, the effective mixing of photons and paraphotons [9], and is phenomenologically equivalent to a particular case of the two-photon electrodynamics earlier studied by Okun [10]. The mass of the V_μ field breaks the conformal invariance of the action and makes possible, in principle, the inflationary amplification of vector fluctuations.

Quite irrespective of the physical (supersymmetric and/or higher-dimensional) origin of the mixing, and of the particular mechanism of mass generation for the graviphoton, the aim of this paper is to discuss whether, for phenomenologically allowed values of α and m, the fluctuations of the massive vector can be efficiently amplified (because of the mass) by inflation, and efficiently converted (through the mixing) into electromagnetic fluctuations, strong enough to seed the cosmic magnetic fields observed on (inter)galactic scale. It should be recalled, to this purpose, that if we identify V_μ with the so-called "fifth-force" vector boson [11], coupled to baryon number, then the strength of the mixing is significantly constrained by geomagnetic data and by Cavendish's tests of Coulomb's law in the geophysical mass range [12], and by pion and kaon decays in the nuclear mass range [13]. Independently from the possible sources of V_μ, the mixing is also constrained by the induced photon-graviphoton oscillations in a dielectric [14], and by changes of electromagnetic polarization due to a possible anomalous magnetic moment of the massive vector [15].

In order to discuss the cosmological amplification of the vacuum fluctuations, in a conformally flat metric $g_{\mu\nu}$, it is convenient to use the conformal time η by setting $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$, and to impose the gauge conditions $\partial_\mu A^\mu =$

$0=\partial_\mu V^\mu$. The action (1) then becomes, after integration by part,

$$S=\int \frac{d^3x d\eta}{8\pi}\Bigg[A_\mu \Box A^\mu + V_\mu \Box V^\mu$$
$$-2\alpha A_\mu \Box V^\mu + m^2 a^2(\eta) V_\mu V^\mu\Bigg], \qquad (2)$$

where the vector indices are now raised and lowered with the Minkowski metric $\eta_{\mu\nu}$, and $\Box \equiv \partial_\eta^2 - \nabla^2$ is the flat-space Dalembertian in the conformal time gauge. In this gauge, the coupling to the background geometry only survives in the mass term, through the scale factor $a(\eta)$.

The above action describes the vector mixing of a non-orthogonal combination of the two mass eigenstates, ψ_μ, ϕ_μ, coupled to a time-dependent external field, $a(\eta)$. By setting

$$\begin{pmatrix} A^\mu \\ V^\mu \end{pmatrix} = \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi^\mu \\ \phi^\mu \end{pmatrix} \qquad (3)$$

the action takes indeed the diagonal form

$$S=\int \frac{d^3x d\eta}{8\pi}\left[\psi_\mu \Box \psi^\mu + (1-\alpha^2)\phi_\mu \left(\Box + \frac{m^2 a^2}{1-\alpha^2}\right)\phi^\mu\right], \qquad (4)$$

and leads to the evolution equations

$$(\psi_k^\mu)'' + k^2 \psi_k^\mu = 0, \qquad (5)$$
$$(\phi_k^\mu)'' + \left(k^2 + \frac{m^2 a^2}{1-\alpha^2}\right)\phi_k^\mu = 0. \qquad (6)$$

Here the prime denotes differentiation with respect to η, and we have expanded the fields into Fourier modes, $\nabla^2 \psi_k = -k^2 \psi_k$. Given an initial vacuum fluctuation spectrum, $\delta\psi_k^\mu$, $\delta\phi_k^\mu$, the massless fluctuations evolve freely, unaffected by the background geometry, and only the massive components, with effective mass $\overline{m} = m/\sqrt{1-\alpha^2}$, are amplified. After the amplification we can thus set $\delta\psi_k^\mu = 0$, and the spectrum of the electromagnetic fluctuations, induced by the mixing of eq. (3), turns out to be proportional to the massive vector spectrum, $\delta A_k^\mu = \alpha\delta\phi_k^\mu$. The spectral energy densities, $\rho(k) = d\rho/d\ln k$, are quadratic in the fields and are thus related by:

$$\rho_A(k) = \alpha^2 \rho_\phi(k) = \alpha^2 \rho_V(k). \qquad (7)$$

For a first, indicative estimate of the possibility of seed production we shall now compute the massive vector spectrum $\rho_\phi(k)$, following the standard methods of cosmological perturbation theory [16], and assuming a "minimal" model of cosmological evolution in which the standard radiation era, starting at $\eta=\eta_1$, is preceeded in time by a phase of exponentially expanding, de Sitter-like inflation (there is no need of considering also the transition to the matter-dominated epoch since, for our purpose, it will be enough to evaluate the spectrum at the intergalactic scale $k/a \sim (1\text{Mpc})^{-1} \sim 10^{-14}$Hz, which re-enters the horizon before the matter-radiation equilibrium). Thus we set $a(\eta) = -(H_1\eta)^{-1}$ for $\eta \ll \eta_1$, and $a \sim \eta$ for $\eta \gg \eta_1$. Our final electromagnetic spectrum will be characterized by three free parameters: the graviphoton mass m, the mixing coefficient α, and the inflation scale H_1.

In the initial inflationary phase eq. (6), for each physical polarization modes ϕ_k, in our case reduces to

$$\phi_k'' + \left(k^2 + \frac{\overline{m}^2}{H_1^2\eta^2}\right)\phi_k = 0, \qquad \overline{m} = \frac{m}{\sqrt{1-\alpha^2}}, \qquad (8)$$

and the solution, normalized at $\eta \to -\infty$ to a vacuum fluctuation spectrum [16], can be written in terms of the second kind Hankel functions [17] as

$$\phi_k(\eta) = |\eta|^{1/2} H_\nu^{(2)}(|k\eta|), \quad \nu = \frac{1}{2}\left(1 - 4\frac{\overline{m}^2}{H_1^2}\right)^{\frac{1}{2}}, \ \eta < \eta_1. \qquad (9)$$

We are interested in modes which are non-relativistic ($k < \overline{m}a$) already at the transition epoch η_1, since for higher modes the mass term can be neglected in eq. (6), and there is no significant amplification throughout the whole inflationary epoch. Such modes are defined by the condition $k < k_M$, where $k_M = \overline{m}a_1 = (\overline{m}/H_1)k_1$, and $k_1 = |\eta|^{-1}$ is the limiting mode crossing the horizon just at $\eta = \eta_1$. We shall assume that the graviphoton mass is much smaller than the inflation scale (typically, we may expect $\overline{m} \simeq m \sim 1$ TeV in the context of standard supersymmetry breaking), so that $k_M/k_1 = (\overline{m}/H_1) \ll 1$.

In the subsequent radiation era the mass term $ma(\eta)$ grows linearly in conformal time, and then dominates always better the evolution equation for all non-relativistic modes $k < k_M$. For such modes we can thus approximate eq. (6), in the radiation era, by

$$\phi_k'' + \beta^2\eta^2\phi_k = 0, \qquad \beta = \overline{m}H_1 a_1^2, \qquad k < k_M, \qquad (10)$$

and the general solution can be written in terms of the Hankel functions as

$$\phi_k(\eta) = \eta^{1/2}\left[c_+(k)H_{1/4}^{(2)}\left(\frac{\beta\eta^2}{2}\right) + c_-(k)H_{1/4}^{(1)}\left(\frac{\beta\eta^2}{2}\right)\right],$$
$$k < k_M, \qquad \eta > \eta_1. \qquad (11)$$

Here $c_\pm(k)$ are the so-called Bogoliubov coefficients [16], determining the spectral number distribution of the pair of vector particles produced from the vacuum. They can be computed by matching the two solutions (9) and (11) and their first derivative at $\eta = \eta_1$. Such a matching can be easily performed by exploiting the small argument limit of the Hankel functions (since $|k\eta_1| < k_M/k_1 = \overline{m}/H_1 = \beta\eta_1^2 \ll 1$), and leads to

$$|c_-(k)| \simeq |c_+(k)| \simeq \left(\frac{k}{k_1}\right)^{-\nu}\left(\frac{H_1}{\overline{m}}\right)^{1/4}, \quad k < k_M \quad (12)$$

(here and throughout we will neglect numerical factors of order unit, as we are primarily interested in an order of magnitude estimate of seed production). We may note that $|c_\pm(k)| \gg 1$, signalling the effective entry of non-relativistic modes into the parametric amplification regime.

The spectral energy distribution of the non-relativistic vector fluctuations, amplified by inflation, can now be conveniently expressed in terms of the proper wavenumber $p = k/a$, and referred to the total critical energy density $\rho_c = 3M_P^2H^2/8\pi$ (M_P is the Planck mass) as

$$\Omega_\phi(p,t) \equiv \frac{p}{\rho_c}\frac{d\rho_\phi}{dp} \simeq \frac{\overline{m}p^3|c_-(p)|^2}{M_P^2H^2} \simeq$$
$$\left(\frac{H_1}{M_P}\right)^2\left(\frac{\overline{m}}{H_1}\right)^{\frac{1}{2}}\left(\frac{H_1}{H}\right)^2\left(\frac{a_1}{a}\right)^3\left(\frac{p}{p_1}\right)^2, \; p < p_M, \quad (13)$$

where we have set $\nu = 1/2$ from eq. (9), working in the assumption $\overline{m}/H_1 \ll 1$. Here $p_M = (\overline{m}/H_1)p_1$, and $p_1 = k_1/a_1 = H_1a_1/a$ is the proper momentum scale crossing the horizon at the end of inflation (today $p_1(t_0) \simeq (H_1/M_P)^{1/2}10^{11}$ Hz). During the matter-dominated era, the above spectrum keeps frozen at the value reached at the time of equilibrium, $\Omega_\phi(t_0) = \Omega_\phi(t_{\rm eq})$. On the other hand, using the kinematics of the radiation era, $(H_1/H_{\rm eq})^2(a_1/a_{\rm eq})^3 = (H_1/H_{\rm eq})^{1/2}$. The final energy spectrum of the electromagnetic fluctuations, obtained from the vacuum through the photon-graviphoton mixing, can be finally estimated from eqs. (7) and (13) as follows:

$$\Omega_A(p,t_0) \simeq \alpha^2\left(\frac{H_1}{M_P}\right)^2\left(\frac{\overline{m}}{H_{\rm eq}}\right)^{\frac{1}{2}}\left(\frac{p}{p_1}\right)^2, \; p < p_M. \quad (14)$$

As anticipated, it only depends on the three parameters α, m and H_1 ($H_{\rm eq} \sim 10^{-55}M_P$ is the known value of the curvature scale at the equilibrium epoch).

In order to seed the galactic dynamo [2] we must now require that the above electromagnetic spectrum extends in frequency to include at least the intergalactic scale $P_G(t_0) = (1{\rm Mpc})^{-1} \simeq 10^{-14}$ Hz, namely

$$p_M = p_1(\overline{m}/H_1) \gtrsim p_G. \quad (15)$$

In addition, the fraction r of seed energy density stored in the mode p_G, relative to the CMBR energy density, $\Omega_{\rm CMB}$, at the epoch of galaxy formation, when $a_{\rm gal} \sim 10^{-2}a_0$, must satisfy the (conservative) condition [2]

$$r = \Omega_A(p_G,t_{\rm gal})/\Omega_{\rm CMB}(t_{\rm gal}) \gtrsim 10^{-34}, \quad (16)$$

in order that the produced magnetic fields may be large enough to seed the galactic dynamo. Finally, the graviphoton mass cannot be arbitrarily large, as the perturbation spectrum, integrated over all modes, has to be at least smaller than one to avoid overcritical density, and to avoid a Universe overdominated by the fluctuations of a massive non-relativistic vector. This requires

$$\int_0^{p_M} d\ln\Omega_\phi(p,t_0) \lesssim 1, \quad (17)$$

where $\Omega_\phi = \Omega_A/\alpha^2$.

An explicit computation shows that the lower bound on m from eq. (15) is always weaker than from eq. (16), provided we limit to "realistic" values of the inflation scale, $H_1 \lesssim 10^{-1}M_P$. Assuming that this is indeed the case, we are left with the conditions (16) and (17) which, by referring the mass to the physically interesting TeV scale, can be written explicitly as

$$10^{-11}\frac{(1-\alpha^2)^{\frac{1}{2}}}{\alpha^4}\left(\frac{M_P}{H_1}\right)^2 \lesssim \left(\frac{m}{1{\rm TeV}}\right) \lesssim 10^5(1-\alpha^2)^{\frac{1}{2}}. \quad (18)$$

The above bounds are illustrated in Fig. 1 for three different values of the inflation scale. The allowed region lies below the curve of critical density, and above the lines of seed magnetic energy. The allowed windows close completely for $\log\alpha \lesssim -3.5$ and, in the limit $\alpha \to 1$, for $H_1 \lesssim 10^{-8}M_P$. The standard inflation scale $H_1 \sim 10^{-5}M_P$ is not excluded, but the allowed region is larger for higher scales.

We may note, to this purpose, that de Sitter inflation at a nearly Planckian scale is not necessarily ruled out by graviton production, provided it is preceeded by a phase of superinflationary expansion [18]. In addition, the presence of such a superinflationary phase leaves unchanged the constraints obtained with our previous analysis, if the de Sitter epoch is long enough to amplify the galactic scale p_G, which is the relevant scale in the context of seed production. This may occur even for $H_1 \sim 0.1M_P$ without clashing with obervational data [19,20], provided – as can be easily checked using the results of [18] – $p_G(0.1M_P/\overline{m}) \gtrsim 10^{-7}$Hz, i.e. $(m/1{\rm TeV}) \lesssim 10^8(1-\alpha^2)^{1/2}$, which is always automatically satisfied because of (18).

The vector masses allowed by the cosmological bounds (18) thus range from 10^5 TeV for $\alpha \ll 1$, to the KeV scale for $\alpha \sim 1$ and $H_1 \to 0.1M_P$ (smaller masses are not forbidden by eq. (18), but would require a fine-tuning of α to 1). It becomes now crucial to include in the discussion all existing bounds on the mixing parameter α, for such a range of masses.

Kwown bounds, at present, are mainly referred to a graviphoton interpretation of V_μ as the "fifth-force" vector, coupled to baryon number with gravitational

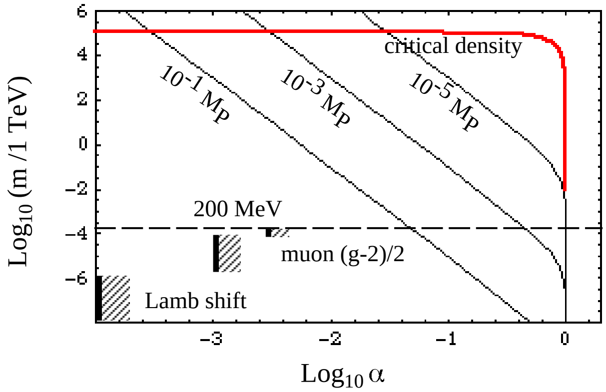

FIG. 1. *Alowed region for the inflationary production of magnetic seeds, in the parameter space of the photon-graviphoton mixing. The allowed region is below the bold curve (critical density), above and to the right of the thin lines (seed magnetic energy, plotted for different H_1), and above the* 200 *Mev dashed line (determined by the anomalous contribution to the $(g-2)/2$ ratio of the muon, by the Lamb shift, and by other particle physics effects not shown in the picture).*

strength. In that case, the mixing is constrained on a macroscopic scale by geo-electric and geo-magnetic data [12], leading to the condition $(m/1\text{TeV}) \gtrsim 10^{-18}\alpha(1-\alpha^2)^{-1/2}$, which is weaker, however, than the lower bound of eq. (18) (except in the limit $\alpha \to 1$, where it forbids arbitrarily small values of the vector mass). A much stronger bound on α would follow [12] from Cavendish's tests of Coulomb's law [21], but it only applies when the graviphoton interaction is active on a macroscopic range ($\lambda \gtrsim 1$ cm, i.e. $m \lesssim 10^{-4}$eV), and then outside the mass range of Fig. 1.

From 10^{-4} eV to 280 MeV there are, however, important bounds on the mixing obtained from nuclear physics [13]. In particular: $\alpha \lesssim 10^{-6}$ for $m = 1.8$ MeV (from beam-dump experiments); $\alpha \lesssim 10^{-4}$ for $m \leq 1$ MeV (from the Lamb shift in hydrogen atoms); $\alpha \lesssim 10^{-3}$ up to $m = 100$ MeV, and $\alpha \lesssim 10^{-2.5}$ for $m = 200$ MeV (from the anomalous magnetic moment of the muon). According to these bounds, all masses smaller than 200 Mev are associated to a mixing parameter α too small to be compatible with the constraints (18), and are thus to be excluded from the allowed region of parameter space (see Fig.1, where we have only reported the most significant bounds).

Summing up the above results, we may conclude that the mixing of the photon with a massive component of the gravitational supermultiplet, typical of extended supersymmetric models, seems to provide an efficient mechanism for the inflationary generation of magnetic seeds for the galactic dynamo. A first estimate of the allowed region in parameter space shows that such a mechanism may be effective for a quite large mass window, 200 MeV – 10^5 Tev. Higher inflation scales (typical, for instance, of string cosmology [6]) seem to be favoured. However, the standard inflation scale $H_1 \sim 10^{-5} M_P$ is also compatible with such a mechanism: the required value of the mixing parameter has to be moderately large, from $10^{-1.5} \simeq 0.03$ to 1, but always compatible with existing phenomenological bounds and theoretical constraints.

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